

Soliton Formation in Chiral Quark Models*

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Abstract

We describe how the non-local regularization can be implemented in the calculation of solitons in the Nambu Jona-Lasinio model as well as in the equivalent linear σ -model. We investigate different forms of regulators and show that the 3-momentum cut-off leads to serious conceptual difficulties.

1 Motivation

This work was done together with Georges Ripka and Wojciech Broniowski.

Solitons corresponding to baryons have been found in several chiral quark models. Many of these solutions turn out to be unstable against collapse unless additional constraints are introduced in the model. The well known examples are the linear NJL model with proper time regularization [1, 2] and the linear σ -model with sea quarks [3, 4]. Even in the linear σ -model with only valence quarks the energy of the soliton becomes too low for any choice of model parameters if one goes beyond the mean field approximation. In all these models the instability occurs because it is energetically favorable for the chiral field to acquire arbitrary (or very) high gradients. This suggests that cutting off high momenta in the interaction may prevent the collapse and stabilize the soliton. A simple sharp cut-off does not yield a stable solution while a smooth behavior of the regulator (usually interpreted as a k -dependent quark mass) can indeed lead to solitons which are stable against the decay into free quarks as well as against collapse. Such a regularization has a physical justification in QCD calculations of the quark propagation in an instanton liquid which predict a non-local effective interaction between quarks with a 4-momentum cut-off $\Lambda \sim 600$ MeV [5].

Further physical implications of the non-local regularization are discussed in the contributions to this workshop by George Ripka and Wojciech Broniowski [6].

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2 The NJL model with non-local regulators

The non-local regularization of the quark-quark interaction can be implemented in the NJL type models by replacing the contact term $(\bar{q}(x)\Gamma_a q(x))^2$, $\Gamma_a \equiv (1, i\gamma_5\tau_a)$ by a non-local form. Usually one introduces a regulator r diagonal in 4-momentum space such that $q(x) \rightarrow \int d_4y \langle x|r|y \rangle q(y)$. The QCD derivation of the quark propagation in a dilute instanton gas predicts the following functional dependence for $r(k^2) = \langle k'|r|k \rangle \delta(k - k')$ [5]:

$$r = f(z) = -z \frac{d}{dz} (I_0(z) K_0(z) - I_1(z) K_1(z)) , \quad z = \frac{\sqrt{k^2} \rho}{2} , \quad (1)$$

where ρ is the instanton size of the order $(600 \text{ MeV})^{-1}$. As we shall see in the following it is necessary to analytically continue the regulator to negative k^2 in order to be able to treat the valence orbit. This is not possible with the form (1) since it has a cut along the negative real axis starting at $k^2 = 0$. We use instead a Gaussian shape of the regulator:

$$r(k^2) = e^{-\frac{k^2}{2\Lambda^2}} , \quad (2)$$

or a “monopole” shape:

$$r(k^2) = \frac{1}{1 + \frac{k^2}{\Lambda^2}} , \quad (3)$$

which has the proper behavior for large k^2 where one gluon exchange dominates.

The expression for the energy of the soliton and the self-consistency equations can be derived from the bosonized Euclidean action

$$I = -\text{Tr} \log (-i\partial_\mu \gamma_\mu + m + r(S + i\gamma_5 P_a \tau_a) r) + \frac{1}{2G^2} \int d_4x (S^2 + P_a^2) , \quad (4)$$

where S and P_a are the chiral fields and are the dynamical variables of the system.

The main difficulty is the presence of time in the regulator. In order to evaluate the trace in (4) it is convenient to introduce energy dependent basis states, which are solutions of the Dirac equation:

$$h(\nu^2) |q_{j\nu}\rangle = e_j(\nu^2) |q_{j\nu}\rangle \quad (5)$$

with

$$h(\nu^2) = -i\vec{\alpha} \cdot \nabla + \beta r(\nu^2 - \vec{\nabla}^2) (S(\vec{r}) + i\gamma_5 \tau_a P_a(\vec{r})) r(\nu^2 - \vec{\nabla}^2) + \beta m . \quad (6)$$

From (4) the following expression for a stationary configuration can be derived [7]:

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \nu d\nu \sum_j \frac{i + \frac{de_j}{d\nu}}{i\nu + e_j(\nu^2)} + \frac{1}{2G^2} \int d_3x (S^2 + P_a^2) - \text{vacuum} . \quad (7)$$

Note that when no regulator (i.e. $r \equiv 1$) or a time-independent regulator is used, the energies e_j are independent of ν and the integration can be carried out using the Cauchy theorem. Closing the contour from below yields the well known expression for the energy of the Dirac sea: $E^{\text{sea}} = \sum_{e_j < 0} e_j$. (Note that the energies of occupied orbits lie on the negative imaginary ν -axis.)

When the soliton describes a baryon, the energy of three valence quarks is added to the energy of the Dirac sea.¹ The same result can be formally obtained by deforming the contour in (7) in such a way as to encircle the valence orbit (for detailed discussion on this point see Wojciech Broniowski contribution to this workshop). Such a prescription gives the expected result provided the orbits do not depend on ν . However, when the regulator depends on time (or ν), this may not lead to the correct result since the regulator generates additional poles scattered in the whole complex ν -plane. It may still work well for an isolated pole on the positive imaginary axis close to 0 as is the case of the 0^+ orbit in the soliton with the hedgehog form of the background chiral field [8]. This pole can then be treated separately, yielding the valence contribution to the soliton energy $E^{\text{val}} = 3e_{\text{val}}$, where the energy of the valence orbit is determined from

$$i\nu + e_{0^+}(\nu^2) \Big|_{\nu^2 = -e_{\text{val}}^2} = 0 . \quad (8)$$

The soliton energy can now be written as:

$$E_{\text{sol}} = E^{\text{val}} + E^{\text{sea}} + E^{\text{meson}} . \quad (9)$$

The sea contribution is

$$E^{\text{sea}} = -N_c \sum_{j \in \text{all}} \int_0^\infty \frac{d\nu}{\pi} \left[\frac{e_j(\nu^2)(e_j(\nu^2) - 2\nu^2 b_j(\nu^2))}{\nu^2 + e_j(\nu^2)^2} - \frac{e_j^0(\nu^2)(e_j^0(\nu^2) - 2\nu^2 b_j^0(\nu^2))}{\nu^2 + e_j^0(\nu^2)^2} \right] \quad (10)$$

with $b_j(\nu^2) = \partial e_j(\nu^2) / \partial \nu^2$ and is evaluated by direct numerical integration along the real ν -axis. The term E^{meson} is given by the last integral in (7) (with the integrand $S^2 + P_a^2 - M_0^2$).

The above prescription is further supported by the fact that it gives an exact result for the baryon number, which can be expressed as [7]:

$$B = -\frac{1}{2\pi i N_c} \int_{-\infty}^\infty d\nu \sum_j \frac{i + \frac{de_j(\nu)}{d\nu}}{i\nu + e_j(\nu)} . \quad (11)$$

The self-consistent equations derived from (4) take the form (the hedgehog ansatz, $P_a(r) = \hat{r}_a P(r)$, for the pion field is assumed):

$$\begin{aligned} \begin{Bmatrix} S(r) \\ P(r) \end{Bmatrix} &= G^2 \left[N_q \text{res}_v^{-1} \tilde{q}_{v\nu 0}^\dagger(\vec{r}) \begin{Bmatrix} \beta \\ i\beta\gamma_5\tau_a\hat{r}_a \end{Bmatrix} \tilde{q}_{v\nu 0}(\vec{r}) \right. \\ &\quad \left. + N_c \int_0^\infty \frac{d\nu}{\pi} \sum_j \frac{e_j(\nu^2)}{\nu^2 + e_j(\nu^2)^2} \tilde{q}_{j\nu}^\dagger(\vec{r}) \begin{Bmatrix} \beta \\ i\beta\gamma_5\tau_a\hat{r}_a \end{Bmatrix} \tilde{q}_{j\nu}(\vec{r}) \right] , \end{aligned} \quad (12)$$

where $\tilde{q}_{j\nu}(\vec{r}) = r((\nu^2 - \vec{\nabla}^2)/\Lambda^2)q_{j\nu}(\vec{r})$ and $\text{res}_v = 1 - i\frac{de_{v\nu 0}}{d\nu}$ is the residue of the valence pole.

A necessary condition for a stable soliton configuration is that the energy (7) is lower than the energy of three free quarks. When the regulator depends on time, the free quark

¹Only if it is positive, otherwise it is already contained in the above sum.

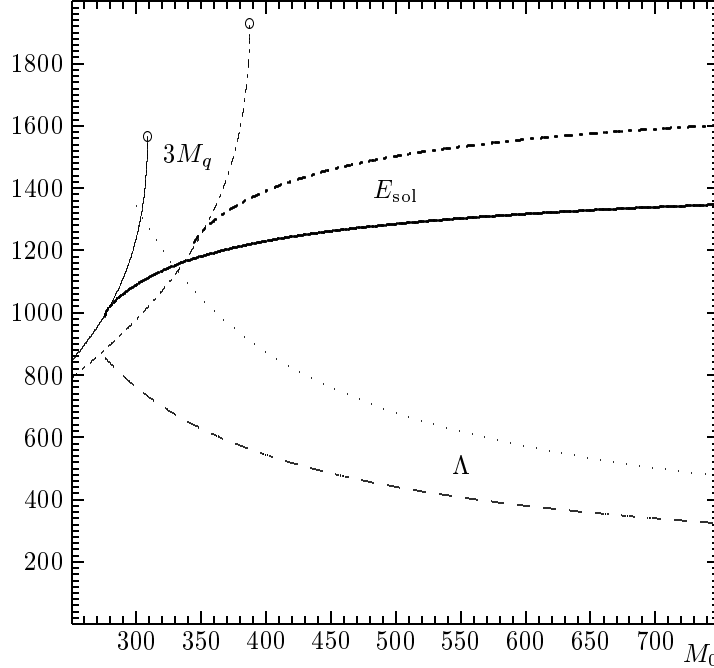


Figure 1: The energy (in MeV) of the soliton (E_{sol}), 3 times the free-space quark mass ($3M_q$) and the cut-off Λ plotted as functions of the parameter M_0 (in MeV). Two parameterizations of the Gaussian regulator are compared; (i) solid and dashed lines: Λ is fitted to $f_\pi = 93$ MeV, (ii) lines containing dots: Λ is fitted to $f_\pi = 1.25 \times 93$ MeV. For the first parameterization $E_{\text{sol}} < 3M_q$ at $M_0 = 276$ MeV and the soliton becomes stable; free quarks do not exist beyond $M_0 = 309$ MeV. For the second parameterization the corresponding values of M_0 are 345 MeV and 387 MeV.

mass, M_q , is not simply the vacuum value of the chiral field, M_0 , but is determined by the position of the pole of the quark propagator in the vacuum [9], *i.e.* it corresponds to the solution of $k^2 + (r(k^2)^2 M_0 + m)^2|_{k^2 = -M_q^2} = 0$. The solution for real k^2 exists only below a critical value of M_0 (see Figure 1); above this point no stable free quarks exist. However, a stable solution can always be found beyond this point provided the quarks dress in a spatially non-uniform background chiral field.

3 Dependence on the shape of the regulator

The model (4) possesses 4 parameters: the vacuum value of the chiral field M_0 , the current quark mass m , the coupling constant G and the cut-off parameter Λ . The coupling constant G is fixed from the stationarity condition in the vacuum, Λ is fitted to the pion decay constant f_π , and m to m_π . We are left with one free parameter M_0 .

In this section we analyze how different shapes of the regulator affect the result. We compare the Gaussian (2), the monopole (3) and a modified version of the instanton (1) regulator. As we have mentioned it is not possible to continue (1) to negative k^2 , which is needed in order to obtain the valence contribution. We have instead introduced a form

which is identical to the instanton regulator for $k^2 > 0$ while its behavior for $k^2 < 0$ is replaced by a real function which approximately follows the real part of (1) for small negative k^2 (“extended instanton”).

The Gaussian and the monopole shapes lead to practically identical results. We have also tried other shapes and found very similar results. The reason is that several properties (including the integral that determines f_π) depend mostly on the behavior of the regulator for small value of k^2 . If this behavior is $r(k^2) \approx 1 - ak^2 + \dots$ then a is almost uniquely determined by the value of f_π . Hence, all shapes with this type of behavior lead to very similar results. The situation is quite different if $dr(k^2)/dk^2 = 0$, *e.g.* for regulators that depend only on k^4 . We do not find stable solutions for such regulators; the energy of the Dirac sea is always higher than the gain due to the lowering of the valence energy.

The form (1) has neither of the above behaviors for $k^2 \sim 0$ but behaves as $1 + \frac{3}{16} \frac{k^2}{\Lambda^2} \ln \frac{k^2}{\Lambda^2}$. It is therefore interesting to check whether stable solitons can also be obtained for such a particular form. We indeed find solitons with very similar properties to those obtained using (2) or (3).

Since f_π sets the scale, it is also interesting to study how a higher value for f_π would affect the results.

Regulator	M_0 [MeV]	Λ [MeV]	m [MeV]	$\langle \bar{q}q \rangle^{1/3}$ [MeV]	e_{val} [MeV]	E_{sea} [MeV]	E_{sol} [MeV]	$\langle r^2 \rangle^{1/2}$ fm	g_A
Gaussian	350	627	10.4	−200	280	1715	1180	1.04	1.16
Gaussian	450	484	15.9	−174	266	1275	1261	0.96	1.12
monopole	350	834	5.24	−252	275	2201	1176	1.05	1.30
monopole	450	639	7.56	−223	260	1628	1261	0.98	1.28
extanton	350	611	4.77	−260	300	2374	1189	1.05	1.04
Gaussian*	450	759	8.75	−246	336	2121	1458	0.83	1.14

Table 1: Properties of the self-consistent soliton solutions for different shapes of the regulator, “extanton” stands for the extended instanton regulator with $\Lambda = 1/\rho$, Gaussian* means a Gaussian regulator with Λ fitted to $f_\pi = 1.25 \times 93$ MeV, $\langle \bar{q}q \rangle$ is the one-flavor quark condensate.

4 Regulators with a 3-momentum cut-off

The calculation can be made much simpler and the problem of analytic continuation to negative k^2 avoided if we take a regulator that does not depend on time. Then the states do not depend on ν and the integration over ν in the above expressions can be carried out analytically. All quantities still remain finite and the soliton remains stable against collapse. The parameters used are the same as for the 4-momentum regulator. However, the explicit breaking of Lorentz invariance leads to serious problems which we discuss in this section.

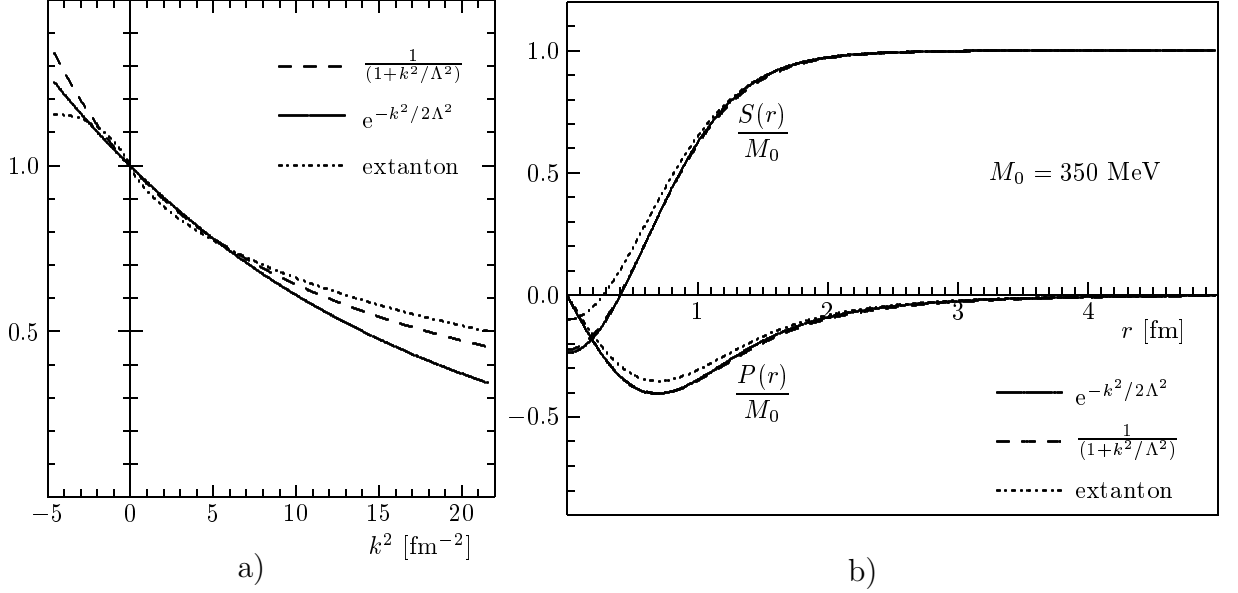


Figure 2: a) Different shapes of regulators for $M_0 = 350$ MeV and Λ fitted to $f_\pi = 93$ MeV, “extanton” stands for the extended instanton regulator with $\Lambda = 1/\rho$. b) Comparison of the self-consistently determined chiral field for the three regulators displayed in a).

The energy of the soliton reduces to

$$E_{\text{sol}} = N_q e_{\text{val}} + \sum_{e_j < 0} (e_j - e_j^0) + \frac{1}{2G^2} \int d_4x (S^2 + P_a^2 - M_0^2) \quad (13)$$

and the self-consistency equation to

$$\begin{aligned} \begin{Bmatrix} S(r) \\ P(r) \end{Bmatrix} &= G^2 \left[N_q q_{\text{val}}^\dagger(\vec{r}) \begin{Bmatrix} \beta \\ i\beta\gamma_5\tau_a\hat{r}_a \end{Bmatrix} r^2 ((-\vec{\nabla}^2)/\Lambda^2) q_{\text{val}}(\vec{r}) \right. \\ &\quad \left. + N_c \sum_{e_j < 0} q_j^\dagger(\vec{r}) \begin{Bmatrix} \beta \\ i\beta\gamma_5\tau_a\hat{r}_a \end{Bmatrix} r^2 ((-\vec{\nabla}^2)/\Lambda^2) q_j(\vec{r}) \right]. \end{aligned}$$

The simplest form of the regulator is a sharp cut-off limiting the 3-momenta to $\vec{k}^2 < \Lambda^2$. Since one usually uses basis states with good $|\vec{k}|$, this is equivalent to restricting the Hilbert space. Such a form is very easy to implement since the regulator does not appear explicitly in the calculation. Unfortunately, no stable soliton solution is found in this case; the contribution of the sea quarks is always larger than the gain due to the lowering of the valence energy.

Stable solutions can be obtained if one takes a smooth form of the regulator, e.g. a Gaussian form $r(\vec{k}^2) = e^{-\vec{k}^2/2\Lambda^2}$. The results are displayed in Figure 3. The threshold value of M_0 below which the soliton does not exist is somewhat larger than for the 4-momentum Gaussian regulator. One notices a rather striking feature that the soliton does not exist *beyond* a certain value of M_0 . The reason for such behavior is the following: the energy of a free quark can be written as $e(|\vec{k}|) = \sqrt{\vec{k}^2 + M_0^2} r^4(\vec{k}^2/\Lambda^2)$. For sufficiently small Λ/M_0

the minimum of $e(|\vec{k}|)$ is not at $|\vec{k}| = 0$ but rather at some $|\vec{k}_0| > 0$, and the energy of a free quark becomes smaller than M_0 . Increasing M_0 the cut-off Λ decreases and at a certain value it becomes energetically favorable for the quarks in the soliton to acquire sufficiently high momenta and leave the soliton. This happens when $3e(|\vec{k}_0|) < E_{\text{soliton}}$. Such an unphysical behavior is a clear consequence of breaking the Lorentz invariance in the interaction.

The value of $|\vec{k}_0|$ can be easily determined if we choose a Gaussian regulator and $m = 0$. For the values of Λ below $\Lambda_c = \sqrt{2}M_0$ the minimum of $e(|\vec{k}|) = \sqrt{\vec{k}^2 + M_0^2}e^{-2\vec{k}^2/\Lambda^2}$ occurs for $\vec{k}_0^2 = \frac{1}{2}\Lambda^2 \ln(2M_0^2/\Lambda^2)$ and the free quark energy at the minimum is: $e(|\vec{k}_0|) = \Lambda\sqrt{\frac{1}{2}(1 + \ln(2M_0^2/\Lambda^2))}$. For $m \neq 0$ this value is slightly modified:

$$e(|\vec{k}_0|) = \Lambda\sqrt{\frac{f_c + \ln \frac{2f_c M_0^2}{\Lambda^2}}{2}}, \quad f_c = \left[\sqrt{1 + \frac{m^2}{2\Lambda^2}} + \frac{m}{\sqrt{2}\Lambda} \right]^2.$$

Figure 3 shows that the soliton exists for $325 \text{ MeV} < M_0 < 600 \text{ MeV}$. At $M_0 = 355 \text{ MeV}$ the cut-off Λ reaches the critical value and the lowest free quark state has $|\vec{k}| > 0$. Nonetheless, the soliton remains bound since $3e(|\vec{k}_0|) > E_{\text{soliton}}$. At $M_0 \approx 600 \text{ MeV}$ the condition is no longer fulfilled and no bound solution exists beyond this point. Between these two values the energy of the soliton differs only little from the energy of three free quarks. The soliton is weakly bound; the chiral field stays close to its vacuum value while the soliton radius is large.

5 Solitons in the equivalent linear σ -model

It is well known that the NJL model can be transformed into the form of an equivalent linear σ -model. This transformation is explained in [9], chapter 5. If one assumes a sharp cut-off and sufficiently large Λ (compared to M_0) the Lagrangian density of the equivalent σ -model takes the familiar form

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi_a)^2 - \frac{\lambda^2}{4} \left(\sigma^2 + \pi_a^2 - f_\pi^2 \right)^2 - \frac{1}{2}m_\pi^2 \left((\sigma - f_\pi)^2 + \pi_a^2 \right). \quad (14)$$

The fields σ and π_a are related to the two components of the chiral field of the bosonized NJL model as $\sigma(x) = S(x)/g$, $\pi_a(x) = P_a(x)/g$, where $g = M_0/f_\pi$. The parameter λ and the mass of the σ meson are related to the parameters of the NJL model by² $g = f_\pi/M_0$, $\lambda^2 = 2g^2$ and $m_\sigma^2 = 4M_0^2 + m_\pi^2$. The question remains whether the above assumptions are met in the model described in section 2.³

We assume that the Lagrangian (14) describes the sea quarks while the valence quarks are treated separately. This is in the spirit of the approaches used in [10, 11, 12, 13, 14].

²The λ^2 here is a factor 2 smaller than the one in [9].

³In fact, they are not; the value of M_0/Λ is close to 1 while the smooth form of the regulator may and does generate additional terms in the Lagrangian (14) and modifies the values of the parameters. A work that will take into account the additional terms is in progress. The conclusions in this section remain valid but the results should be considered only as qualitative.

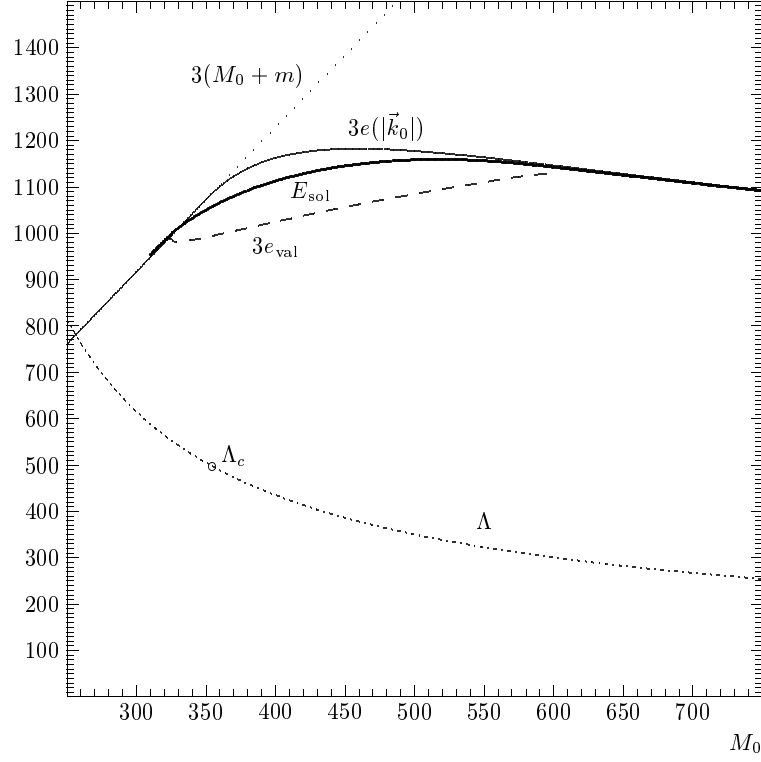


Figure 3: Solutions obtained with a 3-momentum cut-off: the energy (in MeV) of the soliton (bold solid line), 3 times the free quark mass (solid line) and the valence contribution to the soliton energy (dashed line) are plotted as functions of M_0 (in MeV). The energy of the lowest free quark state is equal to $M_0 + m$ only below $M_0 = 355$ MeV, above this value there exist states with $|\vec{k}| > 0$ which have lower energies. The value $|\vec{k}|$ corresponds to the lowest free quark energy. A Gaussian form is used for the regulator, the cut-off Λ (dots) is fitted to $f_\pi = 93$ MeV.

However, in all these approaches the valence orbit is not regularized and – as we shall see in this section – this brings a qualitative difference with respect to the situation where it is regularized. If we agree that a regulator in the quark-quark or, equivalently, quark-chiral field interaction has a well grounded physical origin, the valence orbit should be regularized in the same way as the orbits in the Dirac sea.

The valence orbit is determined as in (8). The soliton energy then becomes ($\sigma' = \sigma - f_\pi$) :

$$E(\sigma, \pi_a) = 3e_{\text{val}} + \int d^3\vec{r} \left(\frac{1}{2} \left[(\nabla\sigma')^2 + m_\sigma^2\sigma'^2 + (\nabla\pi_a)^2 + m_\pi^2\pi_a^2 \right] + g[\sigma j^\sigma + \pi_a j_a^\pi] \right) + E_{s.i.} ,$$

where the meson self-interaction term (the “Mexican hat”) is given by

$$E_{s.i.} = \frac{\lambda^2}{4} \int d^3\vec{r} \left(\sigma'^4 + (\pi_a^2)^2 + 4f_\pi\sigma'^3 + 2\sigma'^2\pi_a^2 + 4f_\pi\sigma'\pi_a^2 \right) .$$

Here we have introduced the source terms which explicitly contain the regulator (see (12)):

$$j^\sigma = N_q \text{res}_v^{-1} q_{v\nu_0}^\dagger(\vec{r}) \beta r^2 ((\nu_0^2 - \vec{\nabla}^2)/\Lambda^2) q_{v\nu_0}(\vec{r}) ,$$

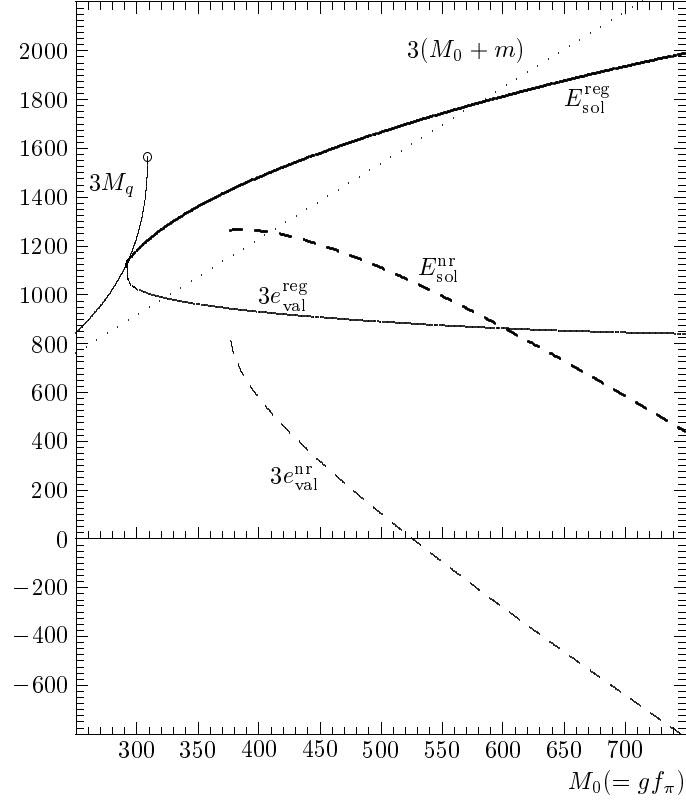


Figure 4: Solitons in linear σ -models: the solid lines show the energy of the soliton (bold), the valence contribution to the energy and of the mass of three free quarks plotted as functions of the parameter M_0 in the case when the valence state is regularized. A 4-momentum Gaussian regulator is used; Λ (see Figure 1) is fitted to $f_\pi = 93$ MeV. The dashed lines show the respective energies when the valence orbit is not regularized and the dotted line represents the corresponding stability line. In this case quasi-stable soliton solutions can exist which never happens in the regularized case.

$$j_a^\pi = N_q \text{res}_v^{-1} q_{v\nu 0}^\dagger(\vec{r}) i\beta\gamma_5\tau_a r^2((\nu_0^2 - \vec{\nabla}^2)/\Lambda^2) q_{v\nu 0}(\vec{r}) . \quad (15)$$

The mean-field solution is obtained by solving the self-consistency equations (the hedgehog ansatz $\sigma(\vec{r}) = \sigma(r)$ and $\pi_a(\vec{r}) = \hat{r}_a\pi(r)$ is assumed):

$$\begin{aligned} \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - m_\sigma^2 \right) \sigma'(r) &= j^\sigma(r) + \lambda^2 \left[(\sigma'(r) + 3f_\pi) \sigma'(r)^2 + (\sigma'(r) + f_\pi) \pi(r)^2 \right] , \\ \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{2}{r^2} - m_\pi^2 \right) \pi(r) &= j_a^\pi(\vec{r}) \hat{r}_a + \lambda^2 \left[\sigma'(r)^2 + 2f_\pi \sigma'(r) + \pi(r)^2 \right] \pi(r) . \end{aligned} \quad (16)$$

In Figures 4 and 5 we compare the properties of the soliton when the valence orbit is regularized and when it is not. Close to the threshold the two solutions do not differ much – as it should be – since here the cut-off Λ is relatively large. For higher values of M_0 (or g) the behavior is qualitatively quite different; in particular, the energy of the unregularized valence orbit soon becomes negative which never happens for the regularized one.

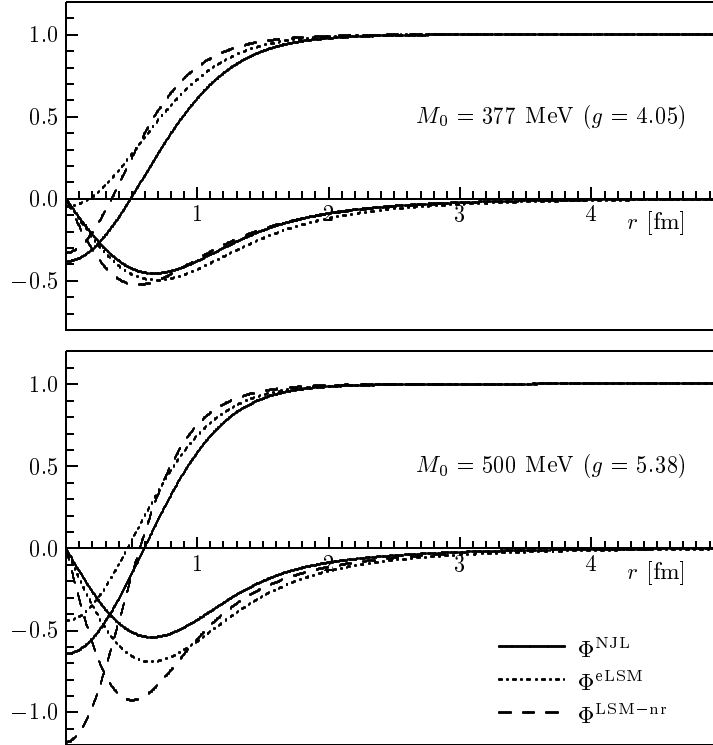


Figure 5: Comparison of the self-consistently determined fields in the NJL model (Φ^{NJL}) and in two versions of the linear σ -model: with (Φ^{eLSM}) and without ($\Phi^{\text{LSM-nr}}$) regularization of the valence state. A Gaussian regulator is used with Λ fitted to $f_\pi = 93$ MeV. For $M_0 = 377$ MeV, the corresponding Λ is relatively large (578 MeV) and the three (pairs of) curves do not differ considerably; for $M_0 = 500$ MeV, Λ is much lower (440 MeV) and the differences become more important: the fields in the unregularized case acquire higher gradients and the soliton shrinks.

Let us finally mention why the regularization is important also in the linear σ -model with only valence quarks. Even if there is no instability as in the regularized linear σ -model⁴, one encounters serious difficulties when going beyond the mean field approximation. In the σ -model it is possible to use the Peierls-Yoccoz projection of good linear momentum, spin and isospin in order to obtain physical nucleon states. It turns out that the energy of the soliton after projection is strongly reduced already when the mean-field solution is used for the chiral fields [15]. If in addition one allows a variation of the chiral field profiles, one obtains a solution with an energy considerably lower than the nucleon mass. The energy gain is mostly due to a strongly localized chiral field which lowers the valence energy. Such a strong localization is not allowed when the regulator is used since it puts a physical limit on the gradients of the field (through the source term (15)).

⁴In the regularized model both the meson and the quark degrees of freedom are used in describing the Dirac sea which leads to appearance of an unphysical pole for the σ -propagator [3, 4].

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